Math 70 7.5 Simplifying Complex Fractions (con't) Lesson 23
7.6 Solving Rational Equations.

Objectives

1) Simplify complex rational expressions
   AKA. complex fractions

2) Solve rational equations
   - find LCD
   - multiply every term by LCD
   - cancel all denominators
   - Solve resulting equation
   - reject extraneous solutions

Math 70 **7.5 Simplifying Complex Rational Expressions**

7.6 Solve Rational Equations

Objectives

1) Simplify complex rational expressions which result from a difference quotient
2) Solve rational equations by "clearing the fractions"
    a. Find LCD
    b. Multiply all terms by the LCD
    c. Cancel all denominators
    d. Solve resulting equation
    e. Reject extraneous solutions
3) Solve rational equations by substitution.

Examples

Find and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$, $h \neq 0$ for the given function.

1) $f(x)=\frac{1}{3x}$

Solve. Check by graphing on GC, if possible.

2) $\frac{4x}{5}+\frac{3}{2}=\frac{3x}{10}$

3) $\frac{x+6}{x-2}=\frac{2(x+2)}{x-2}$

4) $\frac{2x}{2x-1}+\frac{1}{x}=\frac{1}{2x-1}$

5) $x^{-2}+19x^{-1}+48=0$

6) $\left(\frac{3}{x-1}\right)^2+2\left(\frac{3}{x-1}\right)+1=0$

Solve for x.

7) $\frac{x+z}{y}=\frac{6x+9y}{z}$

Bonus & extras on the back!

Bonus Challenge problems

Find and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$, $h \neq 0$ for the given function.

8) $f(x)=\frac{8}{x^2}$

Solve.

9) $(4-x)^2-5(4-x)+6=0$

10) $\frac{z}{2z^2+3z-2}-\frac{1}{2z}=\frac{3}{z^2+2z}$

11) $\frac{2x+3}{3x-2}=\frac{4x+1}{6x+1}$

12) $\frac{2}{x-5}+\frac{1}{2x}=\frac{5}{3x^2-15x}$

13) $\frac{2x}{x-3}+\frac{6-2x}{x^2-9}=\frac{x}{x+3}$

14) $\frac{x^2-20}{x^2-7x+12}=\frac{3}{x-3}+\frac{5}{x-4}$

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(1) Find and simplify the difference quotient

$$\frac{f(a+h)-f(a)}{h} \quad \text{for} \quad f(x)=\frac{1}{3x}.$$

$f(a+h)=\dfrac{1}{3(a+h)}$ replace x by (a+h) use parentheses.

$f(a)=\dfrac{1}{3a}$ replace x by a

h is just a variable.

Substitute these three into given expression, called the difference quotient:

$$\frac{\dfrac{1}{3(a+h)}-\dfrac{1}{3a}}{h}$$ ← complex fraction is not simplified.

Method 1: Multiply by LCD $\dfrac{3a(a+h)}{3a(a+h)}=1$

$$=\frac{\dfrac{1}{\cancel{3}\cancel{(a+h)}}\cdot \cancel{3}a\cancel{(a+h)}-\dfrac{1}{\cancel{3}\cancel{a}}\cdot \cancel{3}\cancel{a}(a+h)}{h\cdot 3a(a+h)}$$

$$=\frac{a-(a+h)}{3ah(a+h)}$$ ← dist

$$=\frac{a-a-h}{3ah(a+h)}$$ ← combine

$$=\frac{-\cancel{h}}{3a\cancel{h}(a+h)}$$ ← cancel $\frac{h}{h}$

$$=\boxed{\frac{-1}{3a(a+h)}}$$

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① Method 2: subtract numerators, then divide

$$\frac{1}{3(a+h)} - \frac{1}{3a} \qquad LCD = 3a(a+h)$$

$$\frac{1}{3(a+h)} \cdot \frac{a}{a} - \frac{1}{3a} \cdot \frac{(a+h)}{(a+h)}$$

$$= \frac{a-(a+h)}{3a(a+h)}$$

$$= \frac{a-a-h}{3a(a+h)}$$

$$= \frac{-h}{3a(a+h)}$$

subtract numerator

$$\frac{\frac{-h}{3a(a+h)}}{h}$$

write this fraction bar using ÷ symbol

$$= \frac{-h}{3a(a+h)} \div h$$

$$= \frac{-\cancel{h}}{3a(a+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-1}{3a(a+h)}}$$

Math 60 Review 1) Solve rational equations by clearing fractions by multiplying all terms both sides by the lowest common denominator.

2) Check graph using x-intercept method
3) Solve by substitution.
4) Reject extraneous solutions by checking domains.

2 Solve $\frac{4x}{5}+\frac{3}{2}=\frac{3x}{10}$ algebraically.

step 1: Find LCD = 10.
step 2: Multiply all terms both sides by LCD.
(We can multiply by something not = 1 because it's an equation!)

$$10\cdot\frac{4x}{5}+10\cdot\frac{3}{2}=10\cdot\frac{3x}{10}$$

step 3: Cancel LCD with denominators before multiplying numerators

$$\overset{2}{\cancel{10}}\cdot\frac{4x}{\cancel{5}}+\overset{5}{\cancel{10}}\cdot\frac{3}{\cancel{2}}=\cancel{10}\cdot\frac{3x}{\cancel{10}}$$

$$2\cdot 4x+15=3x$$
$$8x+15=3x$$

step 4: Identify the type of equation

degree 1 = linear $\Rightarrow$ solve by isolating the variable.

degree 2 = quadratic $\Rightarrow$ Solve by factoring, quadratic formula, complete the square, or graphing.

Note: In Math 70 so far, we have only done factoring, so every quadratic equation in 6.5 can be solved by factoring.

step 5: Solve.

$$15=-5x$$
$$\boxed{-3=x}$$

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EXPLORE

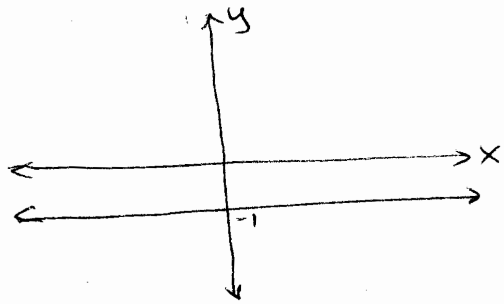
③ Estimate solution of $\frac{x+6}{x-2} = \frac{2(x+2)}{x-2}$ by using the x-intercept graphing method.

step 1: Set equation $= 0$.

$$\frac{x+6}{x-2} - \frac{2(x+2)}{x-2} = 0$$

step 2: Graph in GC being very careful to use added parentheses.

$Y_1 = (x+6)/(x-2) - 2(x+2)/(x-2)$



step 3: Find x-coordinates where graph intersects the x-axis.

None − $\boxed{\text{No solution}}$

DO IT THIS WAY!

③ Solve $\frac{x+6}{x-2} = \frac{2(x+2)}{x-2}$ algebraically.

step 1: Find the domain of each expression by solving equatio made by setting denominator $= 0$

$$x-2=0$$
$$x \neq 2.$$

step 2: Find LCD and multiply all terms on both sides.

$$\frac{\cancel{(x-2)}(x+6)}{\cancel{(x-2)}} = \frac{2(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$x+6 = 2x+4$$
$$2 = x$$

step 3: Reject extraneous solution that's not in domain.

$\cancel{x=2}$ $\boxed{\text{NO SOLUTION}}$

M70

(4) Solve $\frac{2x}{2x-1} + \frac{1}{x} = \frac{1}{2x-1}$ $\quad x \neq 0,\ x \neq \frac{1}{2}$

LCD = $x(2x-1)$

Multiply by LCD

$$\frac{2x}{\cancel{(2x-1)}} \cdot x\cancel{(2x-1)} + \frac{1}{\cancel{x}} \cdot \cancel{x}(2x-1) = \frac{1}{\cancel{(2x-1)}} \cdot x\cancel{(2x-1)}$$

$$2x \cdot x + 1 \cdot (2x-1) = x$$

$$2x^2 + 2x - 1 = x \qquad \text{simplify}$$

$$2x^2 + x - 1 = 0 \qquad \text{set} = 0$$

-2, 2, -1, -1

$$\underbrace{2x^2 + 2x}\ \underbrace{-x - 1} = 0$$

$$2x(x+1) - 1(x+1) = 0$$

$$(x+1)(2x-1) = 0$$

$\boxed{x = -1}$ $\quad \cancel{x = \frac{1}{2}}$ reject extraneous

An <u>extraneous solution</u> is a value found by correct work but which is not in the domain of one or more expressions.

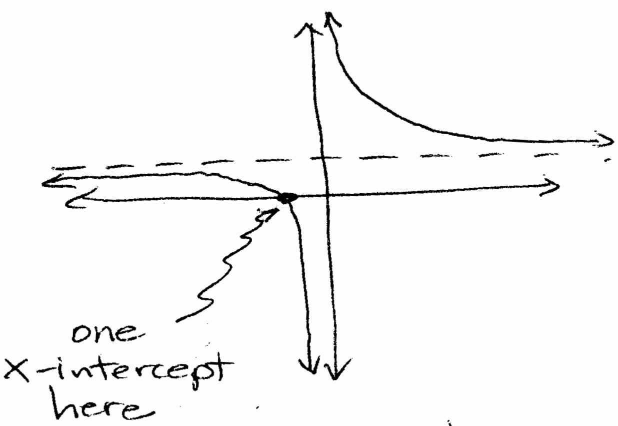
If we substitute this value, we get "undefined" instead of a true statement.

Correct work must reject all extraneous solutions by crossing them out and excluding them from the final answer, including writing "no solution" if all values were rejected.

(4) Solve by graphing

$$\frac{2x}{2x-1} + \frac{1}{x} = \frac{1}{2x-1}$$

$Y_1 = 2x/(2x-1) + 1/x - 1/(2x-1)$



To find by using graphing calculator (TI)

$\boxed{\text{2nd}}\boxed{\text{TRACE}}$ = CALC

2. Zero

Left bound? Move cursor to the left of x-int. $\boxed{\text{ENTER}}$

Right bound? Move cursor to the right of x-int $\boxed{\text{ENTER}}$

Guess? $\boxed{\text{ENTER}}$

$\boxed{x = -1}$ write x-coordinate only.

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Solve by substitution

(5) $x^{-2} - 19x^{-1} + 48 = 0$

Notice $(x^{-1})^2 = x^{-2}$ makes this equation quadratic in form even though it's not a true quadratic.

Substitute $u = x^{-1}$ and factor / solve.

$$u^2 - 19u + 48 = 0$$

$$(u-3)(u-16) = 0$$

$$u = 3 \qquad u = 16$$

48

-3 $\quad$ -16

-19

1, 48
2, 24
3, 16

Replace u by x^{-1} again, solve again.

$$x^{-1} = 3 \qquad x^{-1} = 16$$

$$\frac{1}{x} = 3 \qquad \frac{1}{x} = 16$$

$$1 = 3x \qquad 1 = 16x$$

$$\frac{1}{3} = x \qquad \frac{1}{16} = x$$

$$\boxed{x = \frac{1}{3}, \frac{1}{16}}$$

(6) $\left(\frac{3}{x-1}\right)^2 + 2\left(\frac{3}{x-1}\right) + 1 = 0$ $\qquad$ domain: $x \neq 1$

substitute $u = \frac{3}{x-1}$

$$u^2 + 2u + 1 = 0$$

$$(u+1)(u+1) = 0$$

$$u = -1$$

$$\frac{3}{x-1} = -1$$

$$3 = -(x-1)$$

$$3 = -x + 1$$

$$2 = -x$$

$$\boxed{-2 = x}$$

Math 70 Solving Rational Equations

(6) #6 can be done by two methods:

$$\left(\frac{3}{x-1}\right)^2 + 2\left(\frac{3}{x-1}\right) + 1 = 0$$

Method 1: By substitution

$$u = \frac{3}{x-1} \Rightarrow u^2 = \left(\frac{3}{x-1}\right)^2$$

$$u^2 + 2u + 1 = 0$$

Factor

$$(u+1)(u+1) = 0$$

$$u = -1$$

Substitute back for u:

$$\frac{3}{x-1} = -1$$

Solve for x. Mult by x - 1.

$$(x-1) \cdot \frac{3}{\cancel{(x-1)}} = -1(x-1)$$

$$3 = -x + 1$$

$$2 = -x$$

$$\boxed{x = -2}$$

Check $x \neq 1$ OK

Method 2: By multiplying.

$$\frac{9}{(x-1)^2} + \frac{6}{(x-1)} + 1 = 0$$

$$LCD = (x-1)^2$$

Multiply by LCD:

$$\frac{9\cancel{(x-1)^2}}{\cancel{(x-1)^2}} + \frac{6}{\cancel{(x-1)}} \cdot (x-1)^{\cancel{2}} + 1(x-1)^2 = 0(x-1)^2$$

$$9 + 6(x-1) + (x-1)^2 = 0$$

dist and FOIL

$$9 + 6x - 6 + x^2 - 2x + 1 = 0$$

combine

$$x^2 + 4x + 4 = 0$$

factor

$$(x+2)^2 = 0$$

solve

$$\boxed{x = -2}$$

check

$x \neq 1$ OK

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(7) Solve $\dfrac{x+z}{y} = \dfrac{6x+9y}{z}$ for x

cross-multiply to clear fractions:

$z(x+z) = y(6x+9y)$ dist

$\underline{\underline{x}}z + z^2 = 6\underline{\underline{x}}y + 9y^2$ find the $\underline{\underline{x}}$ values

collect them on same side and other terms to other side

$xz - 6xy = 9y^2 - z^2$

$x(z-6y) = 9y^2 - z^2$ factor out GCF x

$$x = \frac{9y^2 - z^2}{z - 6y}$$

Rational expressions should be left factored

$$\boxed{x = \frac{(3y-z)(3y+z)}{(z-6y)}}$$

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8 $\dfrac{f(a+h)-f(a)}{h}$ for $f(x)=\dfrac{8}{x^2}$

$$\left.\begin{aligned} f(a+h) &= \frac{8}{(a+h)^2} \\ f(a) &= \frac{8}{a^2} \\ h &= \text{just a variable} \end{aligned}\right\}$$

substitute into given expression:

$$\frac{\dfrac{8}{(a+h)^2}-\dfrac{8}{a^2}}{h}$$

Method 1: multiply by LCD: $\dfrac{a^2(a+h)^2}{a^2(a+h)^2}=1$

$$\frac{\dfrac{8}{\cancel{(a+h)^2}}\cdot a^2\cancel{(a+h)^2}-\dfrac{8}{\cancel{a^2}}\cdot \cancel{a^2}(a+h)^2}{h\cdot a^2(a+h)^2}$$

$$=\frac{8a^2-8(a+h)^2}{a^2h(a+h)^2} \leftarrow$$ FOIL $(a+h)^2=a^2+2ah+h^2$

$$=\frac{8a^2-8(a^2+2ah+h^2)}{a^2h(a+h)^2} \leftarrow$$ dist -8

$$=\frac{\cancel{8a^2}-\cancel{8a^2}-16ah-8h^2}{a^2h(a+h)} \leftarrow$$ factor GCF $-8h$

$$=\frac{-8\cancel{h}(2a+h)}{a^2\cancel{h}(a+h)}=\boxed{\frac{-8(2a+h)}{a^2(a+h)}}$$

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(8) Method 2: subtract numerators, then divide

$$\frac{\dfrac{8}{(a+h)^2}-\dfrac{8}{a^2}}{h}$$

Subtract numerators:

$$\frac{8}{(a+h)^2}-\frac{8}{a^2} \qquad LCD = a^2(a+h)^2$$

$$=\frac{8a^2}{a^2(a+h)^2}-\frac{8(a+h)^2}{a^2(a+h)^2}$$

$$=\frac{8a^2-8(a+h)^2}{a^2(a+h)^2} \leftarrow \text{FOIL } (a+h)^2 = a^2+2ah+h^2$$

$$=\frac{8a^2-8(a^2+2ah+h^2)}{a^2(a+h)^2} \leftarrow \text{dist}$$

$$=\frac{\cancel{8a^2}-\cancel{8a^2}-16ah-8h^2}{a^2(a+h)^2} \leftarrow \text{factor } -8h$$

$$=\frac{-8h(2a+h)}{a^2(a+h)^2}$$

Now divide by h:

$$\frac{\left(\dfrac{-8h(2a+h)}{a^2(a+h)^2}\right)}{(h)} \quad \div \text{ symbol}$$

$$=\frac{-8h(2a+h)}{a^2(a+h)^2}\div h \leftarrow \text{multiply by reciprocal}$$

$$=\frac{-8\cancel{h}(2a+h)}{a^2(a+h)^2}\cdot\frac{1}{\cancel{h}} = \boxed{\frac{-8(2a+h)}{a^2(a+h)^2}}$$

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(9) $(4-x)^2 - 5(4-x) + 6 = 0$

$$u = 4-x$$

$$u^2 - 5u + 6 = 0$$

$$(u-2)(u-3) = 0$$

$$u = 2 \qquad u = 3$$

$$4 - x = 2 \qquad 4 - x = 3$$

$$-x = -2 \qquad -x = -1$$

$$\boxed{x = 2 \qquad x = 1}$$

(10) Solve $\dfrac{z}{2z^2+3z-2} - \dfrac{1}{2z} = \dfrac{3}{z^2+2z}$

Note: Lots of twos and z's. This is a deliberate trap for bad handwriting!

Loop the 2's
Cross the z's.
Sharpen z's.
Round the 2's

$$2z^2 + 3z - 2 = 0 \qquad z \neq 0 \qquad z^2 + 2z = 0$$

$$(2z-1)(z+2) = 0 \qquad z(z+2) = 0$$

$$z \neq \tfrac{1}{2}, -2 \qquad z \neq 0, 2$$

$$\text{LCD} = 2z(2z-1)(z+2)$$

$$\frac{z}{\cancel{(2z-1)(z+2)}} \cdot 2z\cancel{(2z-1)(z+2)} - \frac{1}{\cancel{2z}} \cdot \cancel{2z}(2z-1)(z+2) = \frac{3}{\cancel{z}\cancel{(z+2)}} \cdot 2\cancel{z}(2z-1)\cancel{(z+2)}$$

$$2z^2 - (2z-1)(z+2) = 6(2z-1)$$

$$2z^2 - (2z^2 + 3z - 2) = 12z - 6$$

$$2z^2 - 2z^2 - 3z + 2 = 12z - 6$$

$$-3z + 2 = 12z - 6$$

$$8 = 15z$$

$$\boxed{\frac{8}{15} = z}$$

Hint: If you know how to find the LCD and multiply it, but you're not getting the right answer, write out your work with more detail and more neatly.

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(11) $$\frac{2x+3}{3x-2} = \frac{4x+1}{6x+1}$$

$x \neq \frac{2}{3}, -\frac{1}{6}$

$LCD = (3x-2)(6x+1)$

$$\cancel{(3x-2)}(6x+1) \cdot \frac{(2x+3)}{\cancel{(3x-2)}} = \frac{(4x+1)}{\cancel{(6x+1)}} \cdot (3x-2)\cancel{(6x+1)}$$

$$(6x+1)(2x+3) = (4x+1)(3x-2)$$

FOIL!

$$12x^2+18x+2x+3 = 12x^2-8x+3x-2$$

$$20x+3 = -5x-2$$

$$25x = -5$$

$$\boxed{x = -\frac{1}{5}}$$

in domain ✓

(12) $$\frac{2}{x-5} + \frac{1}{2x} = \frac{5}{3x^2-15x}$$

$x \neq 5, 0$ $\qquad 3x(x-5)$

$LCD = 3 \cdot 2 \cdot x \cdot (x-5) = 6x(x-5)$

$$\frac{2}{\cancel{x-5}} \cdot 6x\cancel{(x-5)} + \frac{1}{\cancel{2x}} \cdot \overset{3}{\cancel{6x}}(x-5) = \frac{5}{\cancel{3x}\cancel{(x-5)}} \cdot \overset{2}{\cancel{6x}}\cancel{(x-5)}$$

$$12x + 3(x-5) = 10$$

$$12x + 3x - 15 = 10$$

$$15x = 25$$

$$x = \frac{25}{15}$$

$$\boxed{x = \frac{5}{3}}$$

in domain ✓

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Extras Solve.

(13) $\frac{2x}{x-3} + \frac{6-2x}{x^2-9} = \frac{x}{x+3}$

$x \neq 3$ $(x+3)(x-3)$ $x \neq -3$

$x \neq 3, -3$

LCD $(x+3)(x-3)$

$\frac{2x}{\cancel{x-3}} \cdot (x+3)\cancel{(x-3)} + \frac{6-2x}{\cancel{(x-3)}\cancel{(x+3)}} \cdot \cancel{(x+3)}\cancel{(x-3)} = \frac{x}{\cancel{x+3}} \cdot \cancel{(x+3)}(x-3)$

$2x(x+3) + 6 - 2x = x(x-3)$

$2x^2 + 6x + 6 - 2x = x^2 - 3x$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$\boxed{x = -6, -1}$

all in domain ✓

quadratic (degree 2)
set = 0
factor
set factors = 0.

(14) $\frac{x^2-20}{x^2-7x+12} = \frac{3}{x-3} + \frac{5}{x-4}$

$(x-3)(x-4)$

$x \neq 3, 4$

LCD $= (x-3)(x-4)$

$\frac{x^2-20}{\cancel{(x-3)}\cancel{(x-4)}} \cancel{(x-3)}\cancel{(x-4)} = \frac{3}{\cancel{(x-3)}} \cancel{(x-3)}(x-4) + \frac{5}{\cancel{(x-4)}} (x-3)\cancel{(x-4)}$

$x^2 - 20 = 3(x-4) + 5(x-3)$

$x^2 - 20 = 3x - 12 + 5x - 15$

$x^2 - 20 = 8x - 27$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$\boxed{x = 7, 1}$

all in domain ✓